Int J Theor Phys (2009) 48: 2022–2026
DOI 10.1007/s10773-009-9977-1

Inhomogeneous Plane Symmetric String Cosmological Models in Bimetric Theory

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Received: 26 December 2008 / Accepted: 26 February 2009 / Published online: 12 March 2009

Abstract It is shown that the inhomogeneous plane symmetric perfect fluid distribution and cosmic strings do not survive in frame work of bimetric theory of gravitation proposed by Rosen (Gen. Relativ. Gravit. 4:435, 1973). Hence vacuum models are presented and studied.

Keywords Cosmic strings · Bimetric theory · Perfect fluid

1 Introduction

It is well known that the cosmological models based on General Relativity contain an initial singular state (the big bang) from which the universe expands. The singular state can be avoided if the behaviour of matter and radiation is described by the quantum theory. Unfortunately no body has given a way to do this satisfactorily. A satisfactory physical theory should be free from singularities because the presence of a singularity means a break-down of physical laws provided by the theory. Naturally, taking into consideration these singularities in general relativity one looks carefully at the foundation of general relativity and thinks whether modification can be made to improve it. With this motivation, Rosen [18] proposed a bimetric theory of gravitation incorporating the covariance and equivalence principles. It is based on a simple form of Lagrangian and has a simpler mathematical structure than that of the general theory of relativity. In this theory at each point of space-time, there exist two metric tensors: a Riemannian metric tensor g_{ij} and the background flat space-time metric tensor γ_{ij}. The tensor g_{ij} describes the geometry of a curved space-time and the gravitational fields. Here the background metric tensor γ_{ij} refers to inertial forces. This theory also satisfies covariance and equivalence principles. It is pointed out that this theory agrees with general theory of relativity up to the accuracy of observations made up to now.

The field equations of bimetric theory of gravitation proposed by Rosen [18] are

$$N_j^i - \frac{1}{2}N\delta_j^i = -8\pi\kappa T_j^i \tag{1}$$

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where

$$N^i_j = \frac{1}{2}\gamma^{ab}\left(g^{hi}g_{hj1a}\right)_{1b}$$

and

$$N = N^i_i, \qquad g = \det(g_{ij}), \qquad \gamma = \det(\gamma_{ij}), \qquad \kappa = \left(\frac{g}{\gamma}\right)^{1/2}$$

A vertical bar (1) denotes the covariant differentiation with respect to γ_{ij} and T^i_j is the energy momentum tensor of the matter.

Rosen [18, 19], Yilmaz [25], Karade and Dhoble [7], Karade [8], Israelit [4–6] have studied several aspects of bimetric theory of gravitation. In particular Mohanty and Sahoo [11] and Mohanty et al. [12] have established the non-existence of anisotropic spatially homogeneous Bianchi type cosmological models in bimetric theory when the source of gravitation is governed by either perfect fluid or mesonic perfect fluid. Reddy [13] have discussed the non-existence of anisotropic spatially homogeneous Bianchi type-I cosmological model in bimetric theory of gravitation in case of cosmic strings and Reddy and Venkateswarlu [15] have shown the non-existence of anisotropic Bianchi type-I perfect fluid models in Rosen's bimetric theory. Reddy and Naidu [16] have discussed the non-existence of Kantowski-Sachs cosmological model in cases of string as well as perfect fluid distribution. Reddy et al. [17] have obtained the non-existence of axially symmetric cosmological model with Domain walls and cosmic strings.

The study of string theory has received considerable attention in cosmology. Cosmic strings are important in the early stages of evolution of the universe before the particle creation. Cosmic strings are one dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [24], Latelier [10] and Satchel [22] in general relativity. Relativistic string models in the context of Bianchi space times have been obtained by Krori et al. [9]. Sanyasi Raju and Rao [21] studied Bianchi type-II, VIII & IX in Zero mass scalar fields and self creation cosmology. Bhattacharjee and Baruah [1] studied the problem of cosmic strings taking Bianchi type cosmologies with a self interacting scalar field. Reddy [14] has obtained plane symmetric cosmic strings in Lyra Manifold. Venkateswarlu et al. [23] have investigated Bianchi type-I, II, VIII & IX string cosmological solutions in self creation theory of gravitation. Recently Sahoo [20] have studied spherically symmetric string cosmological models in bimetric theory. Inspite of the fact that a lot of work has been done, in this direction, it is evident from literature that there is need for further investigation which may unravel some of the hidden secrets of the theory.

In this paper it is shown that inhomogeneous perfect fluid distribution and cosmic strings in bimetric theory of gravitation do not survive.

2 Perfect Fluid

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (p+\rho)\,U_iU_j - pg_{ij} \tag{2}$$

together with

$$U^iU_i = -1$$

where U_i is the four velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively.

Consider the inhomogeneous plane symmetric metric in the form

$$ds^2 = E\left(-dt^2 + dz^2\right) + G\left(dx^2 + dy^2\right) \tag{3}$$

where E and G are functions of z and t.

The background flat space-time corresponding to the metric (3) is

$$d\sigma^2 = -dt^2 + dz^2 + dx^2 + dy^2 \tag{4}$$

Using co-moving co-ordinate system the field equations (1) for the metrics (3) and (4) corresponding to the energy momentum tensor (2) in bimetric theory can be written explicitly as

$$\left(\frac{E_3}{E}\right)_3 - \left(\frac{E_4}{E}\right)_4 = 16\pi\kappa p \tag{5}$$

$$\left(\frac{G_3}{G}\right)_3 - \left(\frac{G_4}{G}\right)_4 = 16\pi\kappa p \tag{6}$$

$$\left(\frac{G_3}{G}\right)_3 - \left(\frac{G_4}{G}\right)_4 = -16\pi\kappa\rho \tag{7}$$

where the suffixes 3 and 4hereafter, denote differentiation with respect to z and t respectively.

To solve the field equations (5–7), we note that there are three equations connecting four unknowns, E, G, p and ρ. So one relation connecting these variables is needed. Here, assume the relation between the metric coefficients such as Bhattacharya and Karade [2]

$$E = \alpha G, \quad \alpha \neq 0 \text{ is a constant} \tag{8}$$

From (6) and (7), we have

$$p + \rho = 0 \tag{9}$$

In view of reality conditions, i.e. $p \succ 0$, $\rho \succ 0$, (9) is true only when $p = 0 = \rho$. Thus in bimetric theory the inhomogeneous plane symmetric cosmological perfect fluid model does not survive and hence only vacuum model exists.

Thus the field equations yield

$$\left(\frac{E_3}{E}\right)_3 - \left(\frac{E_4}{E}\right)_4 = 0 \tag{10}$$

By using the method of separation of variables, (10) gives us the solution

$$E(z,t) = \exp\left[\frac{k}{2}\left(z^2 + t^2\right) + k_1 z + k_2 t\right] \tag{11}$$

By the help of (8), we have

$$G(z,t) = \alpha \exp\left[\frac{k}{2}\left(z^2 + t^2\right) + k_1 z + k_2 t\right] \tag{12}$$

where k, k_1 and k_2 are constants.

Thus in view of (11) and (12), the inhomogeneous plane symmetric vacuum model in Rosen's bimetric theory can be written as

$$ds^2 = \exp\left[\frac{k}{2}\left(z^2+t^2\right)+(k_1 z+k_2 t)\right]\left(-dt^2+dz^2\right)$$
$$+\alpha\exp\left[\frac{k}{2}\left(z^2+t^2\right)+(k_1 z+k_2 t)\right]\left(dx^2+dy^2\right) \qquad (13)$$

The model (13) has no singularities either at $z=0$ or at $t=0$ and it does have singularities as $z\to\infty$ and $t\to\infty$. The spatial volume of the model is given by

$$V^3=(-g)^{1/2}=\alpha\exp\left[k\left(z^2+t^2\right)+2\left(k_1 z+k_2 t\right)\right] \qquad (14)$$

which shows that the model is expanding with time t and z since $\alpha\succ 0$.

3 Cosmic Strings

In this section it is established that the inhomogeneous plane symmetric cosmic strings, which have received considerable attention in general relativistic cosmology, do not exist in the frame work of bimetric theory of gravitation proposed by Rosen [18].

Consider the inhomogeneous plane symmetric cosmic string dust source with energy momentum tensor as Latelier [10]

$$T_j^i=\rho U^i U_j-\lambda X^i X_j \qquad (15)$$

where U^i is the four velocity of the string cloud, X^i is the normal space-like four-vector, ρ and λ are the rest energy density of the cloud of strings and the tension density of the string cloud respectively. The string source is along z-axis, which is the axis of symmetry.

Consider the inhomogeneous plane symmetric metric given by (3). Orthonormalisation of U^i and X^i, is given as

$$U^i U_i=-1=-X^i X_i \quad \text{and} \quad U^i X_i=0 \qquad (16)$$

Adopting the co-moving co-ordinate system, the energy momentum tensor (15) take the form

$$T_4^4=-\rho,\qquad T_1^1=0,\qquad T_2^2=0,\qquad T_3^3=\lambda,\qquad T_j^i=0\quad \text{for } i\neq j \qquad (17)$$

The quantities ρ and λ depend on z and t.

Rosen's bimetric field equations (1) for the inhomogeneous plane symmetric metric (3) with the help of (4), (15), (16) and (17) take the form

$$\left(\frac{E_3}{E}\right)_3-\left(\frac{E_4}{E}\right)_4=0 \qquad (18)$$

$$\left(\frac{G_3}{G}\right)_3-\left(\frac{G_4}{G}\right)_4=16\pi\kappa\lambda \qquad (19)$$

$$\left(\frac{G_3}{G}\right)_3-\left(\frac{G_4}{G}\right)_4=-16\pi\kappa\rho \qquad (20)$$

Here, again equations (18)–(20) admit the same solution given by (11) and (12) with

$$\rho = \lambda = 0$$

which in turn yield the same vacuum model in Rosen's theory given by (13). Thus, in bimetric theory of gravitation the inhomogeneous plane symmetric cosmic strings do not survive.

4 Conclusions

It is well known that at early stage of universe cosmic strings play a fundamental role in the formation of universe. It is evident, from literature that Einstein's formalism of general relativity used to establish the existence of cosmic strings. Here it is shown that inhomogeneous plane symmetric perfect fluid distribution and cosmic strings do not survive in Rosen's bimetric theory of gravitation. Hence, bimetric theory doesn't help in any way to study gravitational effects of cosmic strings at the early stages of evolution of the universe.

Acknowledgement The author would like to thank the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing support where the part of this work was carried out.

References

1. Bhattacharjee, R., Baruah, K.K.: J. Pure Appl. Math. **32**, 47 (2001)
2. Bhattacharya, S., Karade, T.M.: Astrophys. Space Sci. **202**, 69 (1993)
3. Gott, J.R.: Astrophys. J. **288**, 422 (1985)
4. Israelit, M.: Gen. Relativ. Gravit. **7**, 623 (1976)
5. Israelit, M.: Gen. Relativ. Gravit. **11**, 25 (1979)
6. Israelit, M.: Gen. Relativ. Gravit. **13**, 681 (1981)
7. Karade, T.M., Dhoble, Y.S.: Lett. Nuovo Cim. **29**, 390 (1980)
8. Karade, T.M.: Indian J. Pure Appl. Math. **11**, 1202 (1980)
9. Krori, K.D., Chaudhury, T., Mahanta, C.R.: Gen. Relativ. Gravit. **26**, 3 (1994)
10. Latelier, P.S.: Phys. Rev. D **28**, 2414 (1983)
11. Mohanty, G., Sahoo, P.K.: Czech. J. Phys. **52**, 1041 (2002)
12. Mohanty, G., Sahoo, P.K., Mishra, B.: Astrophys. Space Sci. **281**, 609 (2002)
13. Reddy, D.R.K.: Astrophys. Space Sci. **286**, 397 (2003)
14. Reddy, D.R.K.: Astrophys. Space Sci. **300**, 381 (2005)
15. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **158**, 169 (1989)
16. Reddy, D.R.K., Naidu, R.L., Rao, V.U.M.: Astrophys. Space Sci. **306**, 185 (2006)
17. Reddy, D.R.K., Naidu, R.L., Rao, V.U.M.: Int. J. Theor. Phys. **47**, 1594 (2008)
18. Rosen, N.: Gen. Relativ. Gravit. **4**, 435 (1973)
19. Rosen, N.: Gen. Relativ. Gravit. **6**, 259 (1975)
20. Sahoo, P.K.: Int. J. Theor. Phys. **47**, 3029 (2008)
21. Sanyasi Raju, Y.S.S., Rao, V.U.M.: Astrophys. Space Sci. **189**, 39 (1992)
22. Satchel, J.: Phys. Rev. D **21**, 2171 (1980)
23. Venkateswarlu, R., Rao, V.U.M., Pavankumar, K.: Int. J. Theor. Phys. **47**, 640 (2007)
24. Vilenkin, A.: Phys. Rev. D **24**, 2082 (1981)
25. Yilmaz, H.: Gen. Relativ. Gravit. **6**, 269 (1975)